HEAT TRANSFER IN A TURBULENT BOUNDARY LAYER WITH STEPPED INJECTION

É. P. Volchkov, V. K. Koz'menko, and V. P. Lebedev

UDC 532.526.4.001.5: 536.242

The asymptotic theory of a turbulent boundary layer has been applied to derive relationships for the heat and mass transfer when there is injection and consequent nonuniformity in the gas composition. Experimental studies are reported on heat and mass transfer with stepped injection of homogeneous and inhomogeneous gases; the results confirm the equations for the heat and mass transfer at a permeable surface when a foreign gas is blown in.

It is usual to examine the heat transfer with gas injection at a nonadiabatic surface (heat flux $q_W \neq 0$) via the hypothesis that the decisive part is played by the difference between the actual wall temperature and the wall temperature under adiabatic conditions ($\Delta T = T_W - T_W^*$); this hypothesis has been confirmed by experiments on heat and mass transfer on impermeable surfaces [1-3]. The results showed that the heat-transfer law used in calculating the boundary layer without injection was conservative. No studies have been made on heat and mass transfer in the injection region on a permeable surface.

The turbulent boundary layer in that case has been examined via the asymptotic theory described by Kutateladze and Leont'ev; it has been shown [4] that the following is the form for the basic limit integral of the theory giving the relative heat-transfer function:

$$\Psi = \frac{M_w^*}{M_0}\frac{T_0}{T_w^*}\left(\int_0^1 \frac{d\omega}{\sqrt{[\psi + (1-\psi)\omega](1+b_1\omega)}}\right)^2 \tag{1}$$

where $\Psi = S/S_0$ is that function, with S and S_0 the Stanton numbers under the working conditions and under standard conditions for the same value of the Reynolds number R_T^{**}, while $\psi = i_W/i_W^*$ is the enthalpy factor for the nonisothermal conditions. $\omega = W/W_0$ is the dimensionless velocity, and b_1 is the wall permeability parameter as calculated from the Stanton number:

$$b_1 = j_w \,/\, \rho_0 W_0 S \tag{2}$$

It follows from (1) that the heat transfer under these conditions is dependent on the parameters of the injection via the molecular weight M_W^* and temperature T_W^* of the gas at the adiabatic wall; integration of (1) gives the effect of the wall permeability and deviation from isothermal conditions on the heat and mass transfer when a foreign gas is injected:

$$\psi < 1,\ \Psi = \frac{M_w^*}{M_0}\frac{T_0}{T_w^*}\frac{4}{(1-\psi)b_1}\left[\ln\frac{\sqrt{(1-\psi)(1+b_1)}+\sqrt{b_1}}{\sqrt{1-\psi}+\sqrt{b_1\psi}}\right]^2 \tag{3}$$

$$\psi > 1,\ \Psi = \frac{M_w^*}{M_0}\frac{T_0}{T_w^*}\frac{4}{(\psi-1)b_1}\left[\operatorname{arc\,tg}\sqrt{\frac{(\psi-1)(1+b_1)}{b_1}} - \operatorname{arc\,tg}\sqrt{\frac{(\psi-1)}{b_1\psi}}\right]^2 \tag{4}$$

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 4, pp. 124-129, July-August, 1973. Original article submitted December 1, 1972.

The critical parameter for the wall permeability b_* (the value of b for which $\Psi \to 0$) can be found from (1) if this is written as

$$\frac{M_w^*}{M_0}\frac{T_0}{T_w^*}\left(\int_0^1 \frac{d\omega}{\sqrt{[\psi+(1-\psi)\omega](\Psi+b\omega)}}\right)^2 = 1 \tag{5}$$

where $b = j_w/\rho_0 W_0 S_0$ is the permeability parameter as calculated from the Stanton number under standard conditions. Integration of (5) with $\Psi = 0$ and $b = b_*$ gives the relationships for the critical injection parameter:

$$\psi < 1, \quad b_* = \frac{M_w^*}{M_0}\frac{T_0}{T_w^*}\frac{1}{1-\psi}\left(\ln\frac{1+\sqrt{1-\psi}}{1-\sqrt{1-\psi}}\right)^2 \tag{6}$$

$$\psi > 1, \quad b_* = \frac{M_w^*}{M_0}\frac{T_0}{T_w^*}\frac{1}{\psi-1}\left(\arccos\frac{2-\psi}{\psi}\right)^2 \tag{7}$$

The heat and mass transfer functions of (3) and (4) can be approximated via the simpler relationship

$$\Psi = \Psi_T\Psi_b = \frac{M_w^*}{M_0}\frac{T_0}{T_w^*}\left(\frac{2}{\sqrt{\psi}-1}\right)^2\left(1-\frac{b}{b_*}\right)^2 \tag{8}$$

where Ψ_T is the heat-transfer function taking into account only the deviation from isothermal conditions, while Ψ_b takes into account only the injection.

These relationships for the transfer differ from analogous relationships in the absence of injection [5-7] by the factor $M_W^* T_0/M_0 T_W^*$, which takes into account the injection.

We determined by experiment the effects of wall permeability and gas composition change on the heat and mass transfer in the injection zone via stepped injection of the same or different gases into turbulent boundary layers; the tests were done in a subsonic aerodynamic tube with a working channel of rectangular cross section 110× 110× 1300 mm. The lower wall of the working channel was a porous section with three parts for injecting gas; the porous plates in each part were made of stainless steel and were 178×98 mm. The position of the upper wall of the channel was adjusted in such a way that the air flow speed remained constant along the length. A detailed description has been given of the apparatus, together with the systems for measuring the thermal and dynamic quantities [8].

We performed two series of calibration experiments. In the first series, we determined the heat transfer at a constant air injection rate (j_w = const) under conditions close to isothermal (t_W = 37-92°C, t_0 = = 27-37°C); in the second series, we determined the performance of the injection beyond the porous part with injection of air and helium. These calibration runs coincided with the results from analogous studies by others [2, 9, 10], and they showed that one can use this device and method under more complicated conditions.

In the stepped injection tests, the gas was injected in two stages; the length of the first part was 178 mm, while that of the second was 356 mm.

With uniform injection in the two parts, we used a relative air flow rate in the two parts in accordance with

$$\bar{j}_{w_1} = \rho_{w_1}V_{w_1}/\rho_0 W_0 = 6.1\cdot10^{-3},\ 12\cdot10^{-3},\ \bar{j}_{w_2} = \rho_{w_2}V_{w_2}/\rho_0 W_0 = 1.1\cdot10^{-3} - 8.2\cdot10^{-3}$$

while the temperature of the injected air was t_1' = 130°C, t_2' = 20°C; the speed of the main flow was W_0 = 20-40 m/sec, while t_0 was 28-42°C.

The results on the heat transfer in the second part were processed using the adiabatic wall temperature T_w^*, which was found by experiment in the absence of injection and heat transfer in this part. The convected heat flux at the wall was determined from the thermal energy balance at the surface of the porous plate. A correction of 1-8% of the convected heat flux was applied for the radiative heat flux. The experimental values of the Stanton and Reynolds numbers in the second part were derived from

$$S = [j_{w_2}c_{p_2}(T_w - T_2') - q_r]/\rho_0 W_0 c_{p_0}(T_w^* - T_w) \tag{9}$$

$$R_T^{**} = \frac{1}{\mu c_{p_0}(T_w^* - T_w)} \int_{x_0} [j_{w_2} c_{p_0} (T_w^* - T_2') - q_r]\, dx \qquad (10)$$

Here T_w and T_w^* are the wall temperatures in the second part under the actual injection conditions and under adiabatic conditions in the absence of injection, while T' is the temperature of the injected gas, x_0 is the start of the second part, q_r is the radiative heat flux, and subscript 2 denotes parameters in the second part. Equation (10) follows from the integral relationship for the energy of the boundary layer with injection.

In processing the experimental data we took into account the effects of the initial dynamic part on the heat transfer on the porous plate by the method of [8]; the effect on the Stanton number was 10-27%.

We processed the heat-transfer data for the second part with allowance for the deviation from isothermal conditions using (8) and the finite value of R^{**} [5]; Fig. 1 shows the results as $\Psi_b = f(b/b_*)$, as with 1) $\bar{j}_{w1} = 6.08 \cdot 10^{-3}$, $\bar{j}_{w2} = 1.11 \cdot 10^{-3}$; 2) $11.9 \cdot 10^{-3}$, $2.18 \cdot 10^{-3}$; 3) $12.4 \cdot 10^{-3}$, $3.26 \cdot 10^{-3}$; 4) $11.7 \cdot 10^{-3}$, $4.92 \cdot 10^{-3}$; 5) $11.7 \cdot 10^{-3}$, $7.1 \cdot 10^{-3}$; 7) $11.5 \cdot 10^{-3}$, $8.23 \cdot 10^{-3}$. The experiments were compared with (8), which describes the heat transfer under injection conditions from a permeable surface. This relationship in the present case ($M_{w^*} = M_0$, $T_0 \approx T_{w^*}$) takes the form of the ordinary relative heat-transfer function [6]:

$$\Psi = (1 - b / b_*)^2 [2 / (\sqrt{\Psi} + 1)]^2 \qquad (11)$$

The experimental points lie around the curve calculated from (11); the discrepancy between the calculated and experimental results at high injection rates may arise from the large errors in the experimental processing near the critical rates. An analogous discrepancy has been observed in simpler experiments with injection constant along the length [10].

Figure 2 shows the characteristic temperature distribution at the wall with stepped air injection; here we give also the calculated wall temperature provided by integral relationships involving the above relative heat-transfer functions. Theory and experiment agree satisfactorily.

These results for stepped injection of the same gas show that one can use the adiabatic wall temperature even when there is gas injection from a permeable surface.

When the injected gas was different from the same flow, the first part received helium and the second received air; the relative flow rate of the helium was $\bar{j}_{w1} = 1 \cdot 10^{-3}$-$2 \cdot 10^{-3}$, while that of the air was $\bar{j}_{w2} = 1 \cdot 10^{-3}$-$8 \cdot 10^{-3}$. The main flow had $t_0 = 24$-$42°C$, while the injected helium had $t_1' = 132$-$135°C$ and the injected air had $t_2' = 19$-$23°C$.

We processed the experimental results on heat transfer for the second part via the difference between the enthalpies of the gas at an adiabatic impermeable wall and at the wall under the actual conditions ($\Delta i = i_w^* - i_w$); the experimental values of the Stanton and Reynolds numbers were found from

$$S = [j_{w_2} (i_w - i_2') - q_r] / \rho_0 W_0 (i_w^* - i_w) \qquad (12)$$

$$R_i^{**} = \frac{1}{\mu (i_w^* - i_w)} \int_{x_0}^{x} [j_{w_2} (i_w^* - i_2') - q_r]\, dx \qquad (13)$$

The gas enthalpy at the wall i_w was calculated from the measured wall temperatures:

$$i_w = c_{p_w} T_w = [c_{p_0} + (K')_w (c_p' - c_{p_0})] T_w \qquad (14)$$

The helium concentration at the wall $(K')_w$ in the second part appears in (14) via the equation for diffusion near the wall, which involves similarity of the heat and mass transfer processes:

$$(K')_w = (K')_w^* / (1 + b_1) \qquad (15)$$

We substitute (15) into (14) and use (2) and (12) to get a relationship for the gas enthalpy at the wall on the second injection part:

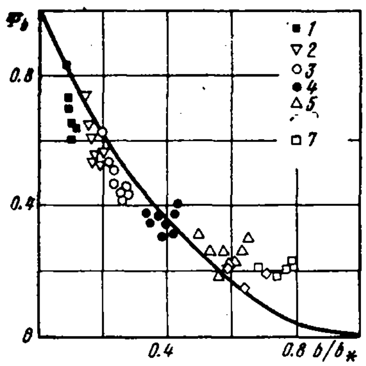

Fig. 1

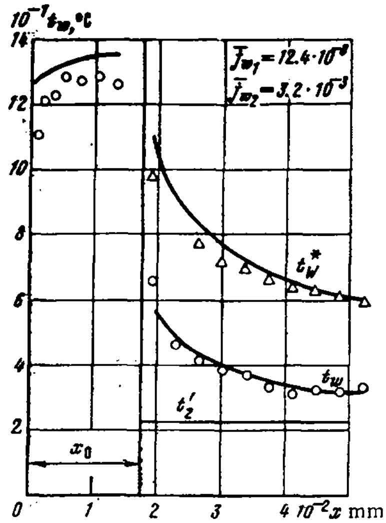

Fig. 2

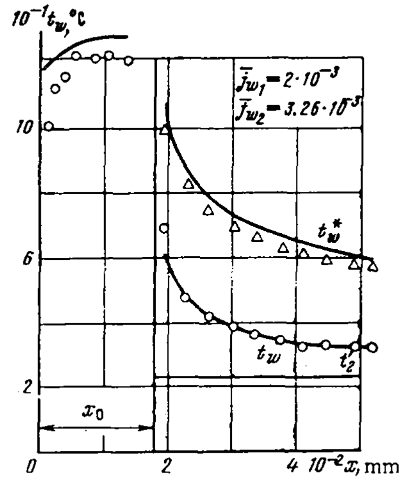

Fig. 3

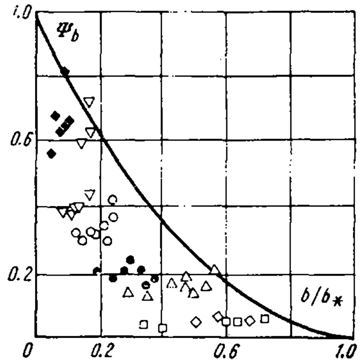

Fig. 4

$$i_w = \frac{c_{p_0} T_w [j_{w_2}(i_w^* - i_2') - q_r] - (K')_w^* (c_p' - c_{p_0}) T_w (j_{w_2} i_2' + q_r)}{[j_{w_2}(i_w^* - i_2') - q_r] - (K')_w^* (c_p' - c_{p_0}) T_w j_{w_2}} \qquad (16)$$

We found the gas enthalpy i_w^* and helium concentration $(K')_w^*$ at the adiabatic wall from the observed values on the injection performance via

$$i_w^* = \theta_i (i_{w_0} - i_0) + i_0 \qquad (17)$$

$$(K')_w^* = \theta_i (K')_{w_0} \qquad (18)$$

where i_{w_0} and $(K')_{w_0}$ are the gas enthalpy and helium concentration at the wall at the end of the first part.

We estimated the effects of thermal diffusion on the heat transfer as about 5% under these conditions. The dynamic part had only a small effect on the heat transfer [8] (5-8% of the Stanton number).

Figures 3-5 give the observed results on the heat and mass transfer with stepped injection as above; Fig. 3 shows the wall temperature variation along the plate in the first and second parts. The experimental results are compared with a calculation performed via the relationships derived for the heat transfer under conditions of foreign gas injection (incorporating $M_w^* T_0/M_0 T_w^*$).

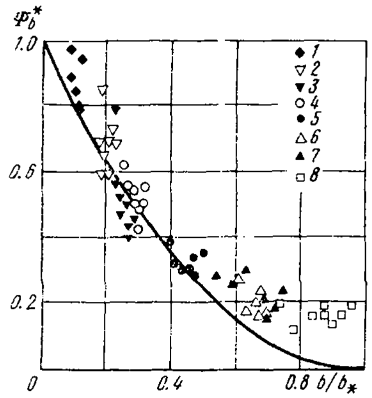

Fig. 5

Figure 4 shows the experimental results as $\Psi_b = S(\sqrt{i_w/i_w^*} + 1)^2/4S_0 = f(b/b_*)$; the values of Ψ_b and b_* were determined in the usual way without incorporating $M_w^* T_0/M_0 T_w^*$, and this processing for injection of air was performed in accordance with (11), but the discrepancy between theory and experiment in that case rose to 300%.

Figure 5 shows the experimental results involving helium injection, where $\Psi_b^* = SM_0T_w^*(\sqrt{i_w/i_w^*} + 1)^2/4S_0M_w^*T_0$; the points correspond to the following injection conditions: 1) $\bar{J}_{w1} = 0.98 \cdot 10^{-3}$, $\bar{J}_{w2} = 1.11 \cdot 10^{-3}$; 2) $1.92 \cdot 10^{-3}$, $2.22 \cdot 10^{-3}$; 3) $1.41 \cdot 10^{-3}$, $3.13 \cdot 10^{-3}$; 4) $2 \cdot 10^{-3}$, $3.26 \cdot 10^{-3}$; 5) $2.03 \cdot 10^{-3}$, $4.92 \cdot 10^{-3}$; 6) $1.9 \cdot 10^{-3}$, $6.81 \cdot 10^{-3}$; 7) $1.35 \cdot 10^{-3}$, $6.97 \cdot 10^{-3}$; 8) $1.86 \cdot 10^{-3}$, $8.21 \cdot 10^{-3}$. The curve has been calculated from (8). We also determined b_* for foreign gas injection from (6), on the basis of the finite Reynolds number; the main effect of the heat transfer in the experiments came from the change in the molecular weight, $M_w^*/M_0 = 0.35$–0.8; the change in the temperature factor T_0/T_w^* was only slight (0.9–0.95). When processed in this way, the experimental results for a stepped injection agree with those for constant injection and with calculations from (8).

These studies on injection show that the heat-transfer coefficient should be determined on the basis of the adiabatic temperature (enthalpy) at the wall; if there is an inhomogeneous boundary layer, one also has to take into account the parameters that appear in the formulas via the factor $M_w^* T_0/M_0 T_w^*$.

LITERATURE CITED

1. É. P. Volchkov, E. G. Zaulichnyi, V. P. Lebedev, A. I. Leont'ev, and E. I. Sinaiko, "An experimental study of the performance of gas injection," in: Heat and Mass Transfer [in Russian], Vol. 1, Énergiya, Moscow (1968).
2. V. P. Komarov and A. I. Leont'ev, "An experimental study of the performance of injection into a turbulent boundary layer of gas," Teplofiz. Vys. Temp., 8, No. 2 (1970).
3. V. P. Lebedev, "An experimental study of a turbulent boundary layer on a smooth plate with stepped heat supply," Zh. Prikl. Mekhan. i Tekh. Fiz., No. 4 (1969).
4. É. P. Volchkov, "Heat and mass transfer in a nonisothermal turbulent flow with injection of another gas," Zh. Prikl. Mekhan. i Tekh. Fiz., No. 5 (1971).
5. Heat and Mass Transfer and Friction in a Turbulent Boundary Layer [in Russian], Izd. AN SSSR, Novosibirsk (1964).
6. S. S. Kutateladze and A. I. Leont'ev, A Turbulent Boundary Layer of Compressible Gas [in Russian], Izd. AN SSSR, Novosibirsk (1962).
7. S. S. Kutateladze, Principles of the Theory of Heat Transfer [in Russian], Nauka, Novosibirsk (1970).
8. É. P. Volchkov, V. K. Koz'menko, and V. P. Lebedev, "Effects of a preceding dynamic part on the heat transfer in a turbulent boundary layer with injection," Zh. Prikl. Mekhan. i Tekh. Fiz., No. 6 (1971).
9. Goldstein, Sheivitt, and Chen, "Performance of film cooling with injection through a porous wall," Proc. Amer. Soc. Mech. Eng. [Russian translation], 87, No. 3 (1965).
10. R. J. Moffat and W. M. Kays, "The turbulent boundary layer on a porous plate: experimental heat transfer with uniform blowing and suction," Internat. J. Heat and Mass Transfer, 11, No. 10 (1968).